Section 8: Summary of Functions

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<table>
<thead>
<tr>
<th>Mathematics Florida Standards will be covered in this section:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A-APR.2.3</strong> - Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</td>
</tr>
<tr>
<td><strong>A-CED.1.1</strong> - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</td>
</tr>
<tr>
<td><strong>A-CED.1.2</strong> - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
</tr>
<tr>
<td><strong>A-CED.1.3</strong> - Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td>
</tr>
<tr>
<td><strong>A-REI.2.4b</strong> - Solve quadratic equations in one variable.</td>
</tr>
<tr>
<td><strong>A-REI.2.4a</strong> - Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as (a \pm bi) for real numbers (a) and (b).</td>
</tr>
<tr>
<td><strong>A-REI.4.11</strong> - Explain why the x-coordinates of the points where the graphs of the equations (y = f(x)) and (y = g(x)) intersect are the solutions of the equation (f(x) = g(x)); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where (f(x)) and/or (g(x)) are linear, rational, absolute value, and exponential functions.</td>
</tr>
<tr>
<td><strong>F-BF.1.1</strong> - Write a function that describes a relationship between two quantities.</td>
</tr>
<tr>
<td><strong>F-BF.1.2</strong> - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
</tr>
<tr>
<td><strong>F-BF.1.3</strong> - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by (f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1)) for (n \geq 1).</td>
</tr>
<tr>
<td><strong>F-BF.2.4</strong> - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
</tr>
<tr>
<td><strong>F-BF.2.5</strong> - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function (h(n)) gives the number of person-hours it takes to assemble (n) engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
</tr>
<tr>
<td><strong>F-BF.2.6</strong> - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
<tr>
<td><strong>F-BF.3.7</strong> - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology in more complicated cases.</td>
</tr>
<tr>
<td><strong>F-BF.3.8</strong> - Identify the effect on the graph of replacing (f(x)) by (f(x) + k), (k f(x)), (f(kx)), and (f(x + k)) for specific values of (k) (both positive and negative); find the value of (k) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</td>
</tr>
<tr>
<td><strong>F-BF.3.9</strong> - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</td>
</tr>
<tr>
<td><strong>F-LE.1.1</strong> - Distinguish between situations that can be modeled with linear functions and with exponential functions.</td>
</tr>
<tr>
<td><strong>F-LE.1.2</strong> - Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</td>
</tr>
<tr>
<td><strong>F-LE.1.3</strong> - Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically or (more generally) as a polynomial function.</td>
</tr>
</tbody>
</table>
Section 8: Summary of Functions
Section 8 – Topic 1
Comparing Linear, Quadratic, and Exponential Functions – Part 1

Complete the table below to describe the characteristics of linear functions.

<table>
<thead>
<tr>
<th>Linear Functions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td></td>
</tr>
<tr>
<td>Rate of Change</td>
<td></td>
</tr>
<tr>
<td>Number of $x$-intercepts</td>
<td></td>
</tr>
<tr>
<td>Number of $y$-intercepts</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
</tbody>
</table>

Sketch the graphs of three linear functions that show all the possible combinations above.

Complete the table below to describe the characteristics of quadratic functions.

<table>
<thead>
<tr>
<th>Quadratic Functions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td></td>
</tr>
<tr>
<td>Rate of Change</td>
<td></td>
</tr>
<tr>
<td>Number of $x$-intercepts</td>
<td></td>
</tr>
<tr>
<td>Number of $y$-intercepts</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
</tr>
<tr>
<td>Domain</td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td></td>
</tr>
</tbody>
</table>

Sketch the graphs of three quadratic functions that show all the possible combinations above.
Complete the table below to describe the characteristics of exponential functions.

<table>
<thead>
<tr>
<th>Exponential Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
</tr>
<tr>
<td>Shape</td>
</tr>
<tr>
<td>Rate of Change</td>
</tr>
<tr>
<td>Number of ( x )-intercepts</td>
</tr>
<tr>
<td>Number of ( y )-intercepts</td>
</tr>
<tr>
<td>Number of vertices</td>
</tr>
<tr>
<td>Domain</td>
</tr>
<tr>
<td>Range</td>
</tr>
</tbody>
</table>

Sketch the graphs of two exponential functions that show all the possible combinations above.

Consider the following tables that represent a linear and a quadratic function and find the differences.

**Linear Function**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

**Quadratic Function**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
</tr>
</tbody>
</table>

How can you distinguish a linear function from a quadratic function?
Consider the following table that represents an exponential function.

<table>
<thead>
<tr>
<th>Exponential Function</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$f(x)$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
</tr>
<tr>
<td>5</td>
<td>243</td>
</tr>
</tbody>
</table>

How can you determine if a function is exponential by looking at a table?

Want some help? You can always ask questions on the Algebra Wall and receive help from other students, teachers, and Study Experts. You can also help others on the Algebra Wall and earn Karma Points for doing so. Go to AlgebraNation.com to learn more and get started!

Section 8 – Topic 2
Comparing Linear, Quadratic, and Exponential Functions – Part 2

Let’s Practice!

1. Identify whether the following key features indicate a model could be linear, quadratic, or exponential.

<table>
<thead>
<tr>
<th>Key Feature</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of change is constant.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>2\textsuperscript{nd} differences, but not 1\textsuperscript{st}, are constant.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has a vertex.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has no $x$-intercept.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has two $x$-intercepts.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Graph has one $y$-intercept.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Domain is all real numbers.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y &gt; 0}$.</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is ${y</td>
<td>y \leq 0}$.</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Range is all real numbers.</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>
Try It!

2. Determine whether each table represents a linear, quadratic, or exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>3</td>
<td>10</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>5</td>
<td>19</td>
<td>6</td>
<td>25</td>
</tr>
</tbody>
</table>

○ Linear ○ Quadratic ○ Exponential
2. Complete the following table so that \( f(x) \) represents a linear function and \( g(x) \) represents an exponential function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Section 8 – Topic 3
Comparing Arithmetic and Geometric Sequences

The founder of a popular social media website is trying to inspire gifted algebra students to study computer programming. He is offering two different incentive programs for students.

**Option 1:** Students will earn one penny for completing their first math, science, or computer-related college course. The amount earned will double for each additional course they complete.

**Option 2:** Students will earn one penny for completing their first math, science, or computer-related college course. For each subsequent course they complete, they will earn \$100.00 more than they did for the previous course.

Write an explicit formula for each option.
Compare the two scholarship options in the tables below.

<table>
<thead>
<tr>
<th>Course</th>
<th>Option 1</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>$0.01</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$0.02</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$0.04</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$0.08</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$0.16</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>$0.32</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>$0.64</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$1.28</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>$2.56</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>$5.12</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>$10.24</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$20.48</td>
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<tr>
<td>13</td>
<td></td>
<td>$40.96</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>$81.92</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>$163.84</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>$327.68</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>$655.36</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>$1,310.72</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>$2,621.44</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>$5,242.88</td>
</tr>
<tr>
<td>21</td>
<td></td>
<td>$10,485.76</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>$20,971.52</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>$41,943.04</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>$83,886.08</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>$167,772.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Course</th>
<th>Option 2</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>$0.01</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$100.01</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$200.01</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$300.01</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$400.01</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>$500.01</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>$600.01</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>$700.01</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>$800.01</td>
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<tr>
<td>10</td>
<td></td>
<td>$900.01</td>
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<tr>
<td>11</td>
<td></td>
<td>$1,000.01</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$1,100.01</td>
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<tr>
<td>13</td>
<td></td>
<td>$1,200.01</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>$1,300.01</td>
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<td>15</td>
<td></td>
<td>$1,400.01</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>$1,500.01</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>$1,600.01</td>
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<td>18</td>
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<td>$1,700.01</td>
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<td>19</td>
<td></td>
<td>$1,800.01</td>
</tr>
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<td>20</td>
<td></td>
<td>$1,900.01</td>
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<td>21</td>
<td></td>
<td>$2,000.01</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>$2,100.01</td>
</tr>
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<td>23</td>
<td></td>
<td>$2,200.01</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>$2,300.01</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>$2,400.01</td>
</tr>
</tbody>
</table>

Compare the two scholarship options in the graphs below.

Option 1 is a geometric sequence.

- Each term is the product of the previous term and two.
- This geometric sequence follows a(n) __________ pattern.
- Evaluate the domain of this function.

Option 2 is an arithmetic sequence.

- Each term is the sum of the previous term and 100.
- Arithmetic sequences follow a(n) __________ pattern.
- Evaluate the domain of this function.

What can be said about the domain of arithmetic and geometric sequences?
Let's Practice!

1. Consider the two scholarship options for studying computer science.
   a. Which scholarship option is better if your college degree requires 10 math, engineering, or programming courses?
   b. What if your degree requires 25 math, engineering, or programming courses?
   c. Do you think that these graphs represent discrete or continuous functions? Justify your answer.
   d. Do you think Option 1 would ever be offered as a scholarship? Why or why not?

Try It!

2. Pablo and Lily are saving money for their senior trip next month. Pablo’s goal is to save one penny on the first day of the month and to triple the amount he saves each day for one month. Lily’s goal is to save $10.00 on the first day of the month and increase the amount she saves by $5.00 each day.
   a. Pablo’s savings plan is an example of a(n)
      o arithmetic sequence.
      o geometric sequence.
   b. Lily’s savings plan is an example of a(n)
      o arithmetic sequence.
      o geometric sequence.
   c. Which person do you think will be able to meet his/her goal? Explain.

3. Circle the best answers to complete the following statement.
   Arithmetic sequences follow a(n) linear | exponential | quadratic pattern, whereas geometric sequences follow a(n) linear | exponential | quadratic pattern, and the domain of both sequences is a subset of the integers | radicals | exponents.
1. On Sunday, Chris and Caroline will begin their final preparations for a piano recital the following Saturday. Caroline plans to practice 30 minutes on the Sunday prior to the recital and increase her practice time by 30 minutes every day leading up to the recital. Chris plans to practice half of Caroline’s time on Sunday, but will double his practice time every day leading up to the recital.

Part A: List Caroline’s and Chris’s practice times on the tables below.

<table>
<thead>
<tr>
<th>Caroline’s Practice Times</th>
<th>Chris’s Practice Times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part B: Compare the graphs of Caroline’s and Chris’s practice times. Identify each graph as linear or exponential.

Want some help? You can always ask questions on the Algebra Wall and receive help from other students, teachers, and Study Experts. You can also help others on the Algebra Wall and earn Karma Points for doing so. Go to AlgebraNation.com to learn more and get started!
Section 8 – Topic 4
Exploring Non-Arithmetic, Non-Geometric Sequences

Let’s review sequences.

Sequences are ____________ whose domain is a subset of the ____________.

We have studied arithmetic and geometric sequences. They can both be defined by explicit and recursive formulas.

Can you think of a sequence that is neither arithmetic nor geometric?

Consider the Fibonacci sequence.

1, 1, 2, 3, 5, 8, 13,...

What pattern do you notice?

Write a recursive formula for the Fibonacci sequence.

Consider the following sequence. Complete the table for the sequence.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What type of function does this sequence appear to be?

What would the domain of this function be restricted to?

Write an explicit formula for this sequence.

Let’s explore how to define a quadratic function with a recursive representation.

We can write a recursive formula as \( f(x) = f(x - 1) \) plus the linear rate of change.

\[
 f(x) = f(x - 1) + mx + b
\]

Use the table above to find the second difference. This will give us a linear rate of change.
Using the second difference as the linear rate of change, write the recursive formula.

\[ f(x) = f(x - 1) + \_\_x + b \]

Now use values from the table to determine \( b \).

Write the recursive formula for the sequence.

Use the recursive formula to find the next two terms of the sequence.

**Let’s Practice!**

1. Consider the following sequence.

   6, 9, 14, 21, 30, ....

   a. What type of function does this sequence appear to be? What are the domain restrictions?

   b. Write the recursive formula for the function.

**Try It!**

2. On an algebra test, Max’s answer was marked incorrect when he was asked to define the recursive formula for the sequence -3, 3, 13, 27, 45, .... Max’s work is shown below. Identify the mistake and correct Max’s work.

\[
\begin{array}{c|c}
 X & Y \\
 1 & -3 \\
 2 & 3 \\
 3 & 13 \\
 4 & 27 \\
 5 & 45 \\
\end{array}
\]

\[
\begin{align*}
f(x) &= f(x-1) + 4x + b \\
f(1) &= f(0) + 4(0) + b \\
13 &= b + 0 + b \\
13 &= 2b \\
b &= 6.5 \\
\end{align*}
\]

\[
f(x) = f(x-1) + 4x + 2
\]
BEAT THE TEST!

1. A recursive formula for a sequence is
   \[ f(x) = f(x - 1) + 6x - 3, \text{ where } f(1) = 3, \]
   which of the following could also represent the sequence?

   A. \( f(x) = \frac{3}{2}x^2, \) where \( x \in \text{ the integers} \)
   
   B. \( f(x) = 3x^2, \) where \( x \in \text{ the integers} \)
   
   C. \( f(x) = \frac{3}{2}x^2, \) where \( x \in \text{ the natural numbers} \)
   
   D. \( f(x) = 3x^2, \) where \( x \in \text{ the natural numbers} \)

---

Section 8 – Topic 5
Modeling with Functions

Let’s discuss the modeling cycle process.

Consider and complete the following diagram that displays the modeling cycle process.

Let’s Practice!

1. The table below represents the population estimates (in thousands) of the Cape Coral-Fort Myers metro area in years since 2010. Employ the modeling cycle to create a graph and a function to model the population growth. Use the function to predict the population in 2020.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>619</td>
<td>631</td>
<td>645</td>
<td>661</td>
<td>679</td>
<td>699</td>
<td>721</td>
</tr>
</tbody>
</table>

   Problem – Identify the variables in the situation and select those that represent essential features.

   a. What are the variables in this situation and what do they represent?
**Section 8: Summary of Functions**

**Formulate** a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.

b. Determine what type of function models the context.

c. Sketch the graph and find the function that models the table.

**Compute** – Analyze and perform operations on these relationships to draw conclusions.

c. Sketch the graph and find the function that models the table.

Population of Cape Coral-Fort Myers Metro Area

- Use the model to predict the population in the year 2020.

**Interpret** the results of the mathematics in terms of the original situation.

e. What do the results tell you about the population growth in Cape Coral-Fort Myers metro area as it relates to the original table?

**Validate** the conclusions by comparing them with the situation, and then either improve the model, or, if it is acceptable, move to the reporting phase.

f. What methods can we use to validate the conclusions?

**Report** on the conclusions and the reasoning behind them.

g. What key elements should be included in your report?
Try It!

2. According to Florida’s Child Labor Law, minors who are 14 or 15 years old may work a maximum of 15 hours per week, and minors that are 16 or 17 years old may work a maximum of 30 hours per week. The relationship between the number of hours that a 15-year old minor in Florida works and his total pay is modeled by the graph below. What is the maximum amount that he can earn in a week?

![Graph showing the relationship between hours worked and total pay.]

Phase 1: _____________

a. Identify the variables in the situation and what they represent.

Phase 2: _____________

b. What type of function can be represented by this graph?

c. Describe the end behavior of the graph.

d. What does the end behavior tell you about the function?

Phase 3: _____________

e. What strategy will you use to create the model for this situation?

f. Find the function of the graph.

Phase 4: _____________

g. Complete the following statement.

The domain that best describes this situation is

\[ \{x \mid x \in \text{rational numbers}\}, \text{ natural numbers}\}, \text{ whole numbers}\} \]

h. What constraints on the domain would exist for a 14-year old? A 17-year old?

i. How much does the student make per hour? Justify your answer algebraically.
Phase 5: ___________

j. Verify that your function accurately models the graph.

k. Are there other ways to validate your function?

Phase 6: ___________

l. What would you report?

**BEAT THE TEST!**

1. Dariel employed the modeling cycle to solve the following problem.

Hannah’s uncle works at the BMW plant in Spartanburg, South Carolina. He purchased a 2017 BMW M2 for Hannah at the manufacturer’s suggested retail price (MSRP) of $52,500. Suppose over the next ten years, the car will depreciate an average of 9% per year. Hannah wishes to sell the car when it is valued at $22,000. When should she sell the car?

When Dariel got to the compute phase, he knew something was wrong. His work is shown below.

---

**Problem:** The variables in the situation are the number of years Hannah has owned the car and the value of the car after a given number of years.

Let $x =$ number of years Hannah has owned the car.

Let $f(x) =$ current value of the car when Hannah has owned it $x$ years.

**Formula:** An exponential function should be used to model the context because the car is depreciating at a common ratio.

**Compute:** The function $f(x) = 52,500(0.9)^x$ models the context where $x$ is the years since 2017 and $f(x)$ is the value of the car. I am going to use a table of values starting at year 2 to try to determine when the car is worth $15,000.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$425.25$</td>
</tr>
</tbody>
</table>

Ugh! This cannot be correct. A 2017 BMW M2 can’t be worth just $425.25 HELP!!!
Part A: Critique his reasoning and give feedback on where he went wrong.

Part B: Complete the modeling cycle.

Section 8 – Topic 6
Understanding Piecewise-Defined Functions

What is a **piecewise function**?

- A function made up of distinct "_________" based on different rules for the ____________.
- The “pieces” of a piecewise function are graphed together on the same coordinate plane.
- The **domain** is the _________________, or the $x$-values.
- The **range** is the ___-values, or output.
- Since it is a function, all “pieces” pass the vertical line test.

Describe an example of a piecewise function used in our daily lives.
Consider the following piecewise-defined function.

\[ f(x) = \begin{cases} 
  x^2 - 2, & \text{when } x \leq 0 \\
  2x + 1, & \text{when } x > 0 
\end{cases} \]

- Each function has a defined _________ value, or rule.
  - \( x \) is less than or equal to zero for the first function.
  - \( x \) is greater than zero for the second function.

- Both of these functions will be on the same graph. They are the “pieces” of this completed piecewise-defined function.

Let’s note some of the features of the graph.

- The domain of the piecewise graph can be represented with intervals. If we define the first interval as \( x \leq 0 \), the second interval would be ____________.

- The graph is nonlinear (curved) when the domain is ________________.

- The graph is linear when the domain is _____________.

- There is one closed endpoint on the graph, which means that the particular domain value, zero, is ___________ in that piece of the function. This illustrates the inclusion of zero in the function ________________.

- There is one open circle on the graph, which means that the particular value, zero, is ___________ in that piece of the function. This illustrates the constraint that \( x > 0 \) for the function ____________.

Label the “pieces” of \( f(x) \) above.
Section 8: Summary of Functions

Let's Practice!

1. Airheadz, a trampoline gym, is open seven days a week for ten hours a day. Their prices are listed below:

   Two hours or less: $15.00
   Between two and five hours: $25.00
   Five or more hours: $30.00

   The following piecewise function represents their prices:

   \[ f(x) = \begin{cases} 
   15, & \text{when } 0 < x \leq 2 \\
   25, & \text{when } 2 < x < 5 \\
   30, & \text{when } 5 \leq x \leq 10 
   \end{cases} \]

   Graph the above function on the following grid.

Try It!

2. Consider the previous graph in exercise 1.

   a. How many pieces are in the step function? Are the pieces linear or nonlinear?

   b. How many intervals make up the step function? What are the interval values?

   c. Why are open circles used in some situations and closed circles in others?

   d. How do you know this is a function?

   e. What is the range of this piecewise function?
BEAT THE TEST!

1. Evaluate the piecewise-defined function for the given values of $x$ by matching the domain values with the range values.

$$f(x) = \begin{cases} 
  x - 1, & x \leq -2 \\
  2x - 1, & -2 < x \leq 4 \\
  -3x + 8, & x > 4 
\end{cases}$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-16</td>
</tr>
<tr>
<td>-5</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

2. Complete the following sentences by choosing the correct answer from each box.

Part A: Piecewise-defined functions are represented by

- one function
- at least one function
- at least two functions

that must correspond to

- different domain values.
- different range values.
- real numbers.

Part B: When evaluating piecewise-defined functions, choose which equation to use based on the

- constant,
- $x$-value,
- slope,

then substitute and evaluate using

- exponent rules.
- order of operations.
- your instincts.

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Section 8 – Topic 7
Absolute Value Functions

Consider the absolute value function: \( f(x) = |x| \).

Sketch the graph of \( f(x) \) by completing the table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Absolute value functions are a special case of a ______________________ function.

Write a different function that represents \( f(x) \).

### Let's Practice!

1. Sketch the graph of \( g(x) = |x| + 1 \).

   \[
   \begin{array}{c|c}
   x & g(x) \\
   \hline
   -2 & -1 \\
   -1 & 0 \\
   0  & 1  \\
   1  & 2  \\
   2  & 3  \\
   \end{array}
   \]

   Write \( g(x) \) as a piecewise-defined function.

2. Consider the function below.

   \[ t(x) = \begin{cases} 
   x + 2, & \text{when } x < -2 \\
   -x - 2, & \text{when } x \geq -2 
   \end{cases} \]

   Write the absolute value function that represents \( t(x) \).
Try It!

3. Sketch the graph \( f(x) = |x - 2| + 3 \).

![Graph of \( f(x) = |x - 2| + 3 \)]

Write \( f(x) \) as a piecewise-defined function.

4. Compare and contrast \( h(x) \) and \( m(x) \).

\[
\begin{align*}
h(x) &= |x - 1| \\
m(x) &= \begin{cases} 
  x - 1, & \text{when } x < 0 \\
  -x - 1, & \text{when } x \geq 0
\end{cases}
\end{align*}
\]

BEAT THE TEST!

1. Consider the following piecewise-defined function.

\[
f(x) = \begin{cases} 
  -x - 3, & \text{when } x < 0 \\
  x - 3, & \text{when } x \geq 0
\end{cases}
\]

Which of the following functions also represent \( f(x) \)?

- \( g(x) = |x - 3| \)
- \( h(x) = |x + 3| \)
- \( m(x) = |x| - 3 \)
- \( n(x) = |x| + 3 \)

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Section 8 – Topic 8
Graphing Power Functions – Part 1

A power function is a function in the form of $f(x) = kx^n$, where $k$ and $n$ represent the set of all real numbers.

A square root function is an example of a ___________ function.

Sketch the graph of $f(x) = \sqrt{x}$ on the set of axes below.

Describe the domain and range.

A ___________ ___________ is a number that multiplies by itself three times in order to create a cubic value. A function is called a cube root function if _____________.

Sketch the graph of $f(x) = \sqrt[3]{x}$ on the set of axes below.

Describe the domain and range.
Let's Practice!

1. Consider the following function.

\[ f(x) = \sqrt{x} + 2 \]

a. Sketch the graph of \( f(x) \) on the set of axes below.

b. Describe the domain and range.

2. Consider the following function.

\[ g(x) = \frac{1}{\sqrt{x}} - 2 \]

a. Sketch the graph of \( g(x) \) on the set of axes below.

b. Describe the domain and range.
Try It!

3. Consider the following functions.

\[ h(x) = \sqrt{x} - 1 \]
\[ m(x) = \frac{3}{x} - 1 \]

a. Sketch the graphs of \( h(x) \) and \( m(x) \) on the set of axes below.

b. Compare and contrast the graphs of \( h(x) \) and \( m(x) \).

A **cubic function** is any function of the form _________________.

Sketch the graph of \( f(x) = x^3 \) on the set of axes below.

Describe the domain and range.

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Let's Practice!

1. Consider the following function.

\[ f(x) = x^3 + 1 \]

a. Sketch the graph of \( f(x) \) on the set of axes below.

b. Describe the domain and range.

Try It!

2. Consider the following functions.

\[ g(x) = x^3 \]
\[ h(x) = \frac{3}{\sqrt{x}} \]

a. Sketch the graphs of \( g(x) \) and \( h(x) \) on the set of axes below.

b. What observations can you make about the relationship between \( g(x) \) and \( h(x) \)?
1. Consider the following functions.

\[ m(x) = \sqrt{x} \]
\[ p(x) = \frac{3}{\sqrt{x}} \]
\[ t(x) = x^3 \]

Which of the following statements is correct about the graph of \( m(x) \), \( p(x) \), and \( t(x) \)?

A. The domain of \( m(x) \), \( p(x) \), and \( t(x) \) is all real numbers.
B. The range of \( p(x) \) and \( t(x) \) is \([3, \infty)\).
C. The range of \( m(x) \) is \([0, \infty)\).
D. \( m(x) \), \( p(x) \), and \( t(x) \) share the points \((0, 0)\), \((1, 1)\), and \((-1, -1)\).

2. A cube's volume, \( V(s) \), is given by the equation \( V(s) = s^3 \), where \( s \) is the length of each side.

Part A: Sketch the graph that models the relationship between the volume and length of each side of a cube.

Part B: What are the domain and range of this relation?
Part C: If we know the volume of a cube, write a function \( s(V) \) that we can use to find the length of the sides of the cube.

Part D: How would the graph of \( s(V) \) compare to the graph of \( V(s) \)?
Consider the following fourth degree polynomial function.

\[ g(x) = x^4 - 4x^2 \]

Find the range of \( g(x) \) for the given domain \( \{-2, -1, 0, 1, 2\} \).

Does the above domain contain zeros of \( g(x) \)? Justify your answer.

Consider the following third degree polynomial function.

\[ h(x) = -x^3 - 5x^2 \]

Find the zeros of the function \( h(x) \).

---

**Let’s Practice!**

1. Consider the following graph of \( f(x) \).

   ![Graph of f(x)](image)

   What are the zeros of \( f(x) \)?

2. What are the zeros of \( g(x) = x(x + 1)(x - 2) \)?
Try It!

3. Consider the function \( h(x) = x^3 - 3x^2 + 2 \).

   a. Find the range of \( h(x) \) given domain \{−1, 1, 3\}.

   b. Are any zeros of \( h(x) \) found in the above domain? Justify your answer.

   c. Consider the graph of \( h(x) \).

   ![Graph of h(x)](image)

   What are the other zeros of \( h(x) \)?

BEAT THE TEST!

1. Which of the graphs has the same zeros as the function \( f(x) = 2x^3 + 3x^2 - 9x \)?

   ![Graph Options](image)

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Section 8 – Topic 11
End Behavior of Graphs of Polynomials

Make observations about following the end behavior of the following graphs.

\[ y = x^2 \]

\[ y = -x^2 \]

\[ y = x^3 \]

\[ y = -x^3 \]
Use your observations to sketch the graphs and make conjectures to complete the table.

### End Behavior of Polynomials

<table>
<thead>
<tr>
<th>End Behavior</th>
<th>Leading Coefficient is Positive</th>
<th>Leading Coefficient is Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f(x) = x^2 )</td>
<td>( f(x) = -x^2 )</td>
</tr>
</tbody>
</table>

**Degree of Polynomial is Even**

\[
\begin{align*}
\text{As } x \to \infty, f(x) & \quad \text{as } x \to -\infty, f(x) \\
\text{As } x \to \infty, f(x) & \quad \text{as } x \to -\infty, f(x)
\end{align*}
\]

**Degree of Polynomial is Odd**

\[
\begin{align*}
\text{as } x \to \infty, f(x) & \quad \text{as } x \to -\infty, f(x) \\
\text{as } x \to \infty, f(x) & \quad \text{as } x \to -\infty, f(x)
\end{align*}
\]

### Let's Practice!

1. Consider the following graph of \( f(x) \).

   ![Graph of f(x)](image)

   a. Does the function \( f(x) \) have an even or odd degree? Justify your answer.

   b. Is the leading coefficient of \( f(x) \) positive or negative? Justify your answer.

2. Describe the end behavior of the function \( g(x) = -5x^3 + 8x^2 - 9x \).
Try It!

3. Consider the following graph of $f(x)$.

![Graph of $f(x)$]

a. Does the function $f(x)$ have an even or odd degree? Justify your answer.

b. Is the leading coefficient of $f(x)$ positive or negative? Justify your answer.

4. Describe the end behavior of the function below.

$$p(x) = \frac{1}{2}x^6 - x^5 - x^4 + 2x^3 - 2x + 2$$

BEAT THE TEST!

1. Determine which of the following statements is true for the function $f(x) = 3x^5 + 7x - 4247$?

   - A. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$
   - B. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$
   - C. As $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$
   - D. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$

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Section 8 – Topic 12
Graphing Polynomial Functions of Higher Degrees

Consider the following function.

\[ g(x) = -(x + 3)(x - 1)(x - 2) \]

Describe the end behavior of the graph of \( g(x) \).

Find the zeros of \( g(x) \).

Use the end behavior and zeros to sketch the graph of \( g(x) \).

Let’s Practice!

1. Sketch the graph of the following polynomial.

\[ f(x) = (x - 2)(x + 3)(x + 5) \]
2. Sketch the graph of the following polynomial.

\[ f(x) = -(x - 5)(x + 4)(3x - 1)(x + 2) \]

Try It!

3. Sketch a graph of the following polynomial.

\[ f(x) = (x - 1)(x + 2)(x - 3)(x + 1) \]
1. Match each equation with its corresponding graph.

A. \( y = (x + 1)(x - 3)(x + 2) \)  
B. \( y = -(x + 1)(x - 3)(x + 2) \)  
C. \( y = -x(x + 1)(x - 3)(x + 2) \)  
D. \( y = x(x + 1)(x - 3)(x + 2) \)

---

Section 8 – Topic 13
Recognizing Even and Odd Functions

An even function has symmetry about the _________.

An odd function has symmetry about the _________.

Consider the following graphs. Label each graph as even, odd, or neither in the space provided.

---

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If a function is even, then \( f(-x) = \_\_\_\_\_. \)

If a function is odd, then \( f(-x) = \_\_\_\_\_. \)

To determine if a function is even or odd:
- Substitute \((-x)\) into the function.
- If the resulting polynomial is the same, then the function is \_\_\_\_.
- If the resulting polynomial is the exact opposite, then the function is \_\_\_\_.
- If the resulting polynomial is neither the same or the exact opposite, the function is not even nor odd.

**Let's Practice!**

1. Complete the table below to determine if the following functions are even, odd, or neither.

<table>
<thead>
<tr>
<th>Function</th>
<th>Value of ( f(-x) )</th>
<th>Even, Odd, or Neither?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^4 + x^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = x^6 + 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 6x^5 - x^3 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Try It!**

2. Give an example of a polynomial function that is an even function.

3. Give an example of a polynomial function that is an odd function.

4. Give an example of a polynomial function that is neither odd nor even.
BEAT THE TEST!

1. Use the following functions to complete the table below.

\[ f(x) = x^2 \]
\[ g(x) = x^3 \]
\[ h(x) = x^3 - 4x \]

<table>
<thead>
<tr>
<th>Function</th>
<th>Even</th>
<th>Odd</th>
<th>Neither</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) \cdot g(x) )</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>( g(x) \cdot h(x) )</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>( f(x) + h(x) )</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>( g(x) + h(x) )</td>
<td>O</td>
<td>O</td>
<td>O</td>
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Section 8 – Topic 14
Solutions to Systems of Functions

Consider the functions \( f(x) \) and \( g(x) \). What can be said about the \( x \)-coordinates where \( f(x) \) intersects \( g(x) \)?

Name two methods of finding the solutions to the system containing \( f(x) \) and \( g(x) \).

Let's Practice!

1. Given the exponential function, \( f(x) = 2^x \), and the cubic function, \( g(x) = x^3 - 2x \), find the positive \( x \) value where \( f(x) = g(x) \) by completing a table of values from \( x = 0 \) to \( x = 4 \).

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<th>( x )</th>
<th>( f(x) )</th>
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2. Given the graphs of an absolute value function, \( f(x) = 4 - |x| \), and a logarithmic function, \( g(x) = \log_3 x \), find the coordinate where \( f(x) = g(x) \).

3. It takes Samara three hours to complete a job. Samara often works with Frederick. The function that models the time that it takes them to complete the job together is given by \( f(x) = \frac{3x}{x+3} \), where \( x \) represents the number of hours it takes Frederick to complete the job alone. The graph of \( f(x) \) is shown below.

   a. Should the domain of \( f(x) \) be restricted?

   b. Sketch the graph of \( x = 3 \) on the same coordinate plane and interpret the meaning of the intersection in the given context.

   c. Sketch the graph of \( x = 7 \) on the same coordinate plane and interpret the meaning of the intersection in the given context.
1. Given the exponential function \( f(x) = 4 \cdot 3^x \) and the polynomial function \( g(x) = x^4 + 5x^2 \), find the coordinate where \( f(x) = g(x) \).

Great job! You have reached the end of this section. Now it’s time to try the “Test Yourself! Practice Tool,” where you can practice all the skills and concepts you learned in this section. Log in to Algebra Nation and try out the “Test Yourself! Practice Tool” so you can see how well you know these topics!