Variables and Patterns UNIT

Main Focus: Focus on Algebra

Variables and Patterns (Relationships): Develop an understanding of variables and how they are related.

- Explore problem situations that involve variables and relationships
- Identify the dependent and independent variable and describe how they are related in a situation
- Interpret the 'stories' told by patterns in tables and coordinate graphs of numeric \((x, y)\) data;
- Represent the pattern of change between two variables in words, data tables, graphs, and equations
- Investigate situations that change over time
- Examine increasing and decreasing patterns of change
- Compare linear and non-linear patterns of change by using tables or graphs
- Use tables, graphs and equations to find the value of a variable given the value of the associated variable
- Explore relationships to become familiar with graphing in all four quadrants
- Describe advantages and disadvantages of using words, tables, graphs and equations to represent patterns of change between two variables and make connections across those representations
- Write an equation to express the relationship between two variables in one and two operations: \(y = mx, y = b + x,\) and \(y = b + mx\)
- Calculate average speed and show how it is reflected in a table or graph and vice versa.
- Recognize and express direct proportionality relationships with a unit rate \((y = mx)\) and represent these relationships in rate tables and graphs
- Solve problems that involve variables

Expressions and Equations: Develop an understanding of expressions and equations.

- Use properties of operations, including the Distributive Property and the Order of Operations, to write equivalent expressions for the dependent variable in terms of the independent variable
- Use tables, graph or properties of numbers such as the Distributive Property to show that two expressions are equivalent
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity
- Interpret and evaluate expressions in which letters stand for numbers and apply the Order of Operations as needed
- Recognize that equations are statements of equivalence between two expressions
- Solve linear equations of the form, \(y = ax, y = b + x,\) and \(y = b + ax\) using numeric guess and check, tables of \((x, y)\) values, graphs or fact families
- Write an inequality and associate it with an equation to find solutions and graph the solutions on a number line
"I Can Translate a Written Mathematical Expression into a Symbolic Expression."
Writing Numerical and Algebraic Expressions and Equations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Key Words</th>
<th>Example</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition (+)</td>
<td>Plus</td>
<td>Seventeen plus thirty-seven</td>
<td>17 + 37</td>
</tr>
<tr>
<td></td>
<td>More than</td>
<td>Ten more than x</td>
<td>x + 10</td>
</tr>
<tr>
<td></td>
<td>Increased by</td>
<td>A number increased by twenty</td>
<td>x + 20</td>
</tr>
<tr>
<td></td>
<td>Added to</td>
<td>Eleven added to forty-five</td>
<td>11 + 45</td>
</tr>
<tr>
<td></td>
<td>SUM</td>
<td>The sum of 20 and j</td>
<td>20 + j</td>
</tr>
</tbody>
</table>

| Subtraction (−) | Minus              | A number minus seven         | n − 7       |
|                 | Decreased by       | Four decreased by m          | 4 − m       |
|                 | Fewer than         | 2 fewer than 10              | 10 − 2      |
|                 | Less than          | Five less than eight         | 8 − 5       |
|                 | Subtracted from    | Some number subtracted from ten | 10 − n      |
|                 | DIFFERENCE         | The difference of 19 and 12  | 19 − 1      |

| Multiplication (⋅) | Times              | Five times a number         | 5 ⋅ n       |
|                   | Twice/Double       | Twice a number / Double a number | 2 ⋅ x       |
|                   | Triple             | Triple a number             | 3 ⋅ a       |
|                   | PRODUCT            | The product of three and twelve | 3 ⋅ 12      |

| Division (÷)      | Divided by         | One hundred divided by 20   | 100 ÷ 20    |
|                   | Divided into       | Six divided into 30          | 30 ÷ 6      |
|                   | Split Into Groups  | 24 split into 4 groups       | 24 ÷ 4      |
|                   | Cut in half        | A number cut in half         | a ÷ 20      |
|                   | QUOTIENT           | The quotient of a number and 2 | q ÷ 2      |

| Power (²)         | Square             | x squared                    | x²          |
|                   | Cube               | 6 cubed                      | 6³          |
|                   | Raised to the Power| t raised to the fifth power  | t⁵          |
Translate the following algebraic expressions.

1. The sum of 15 and \( v \)
\[ 15 + v \]

2. The quotient of a number and 7
\[ x \div 7 \]

3. A number to the eighth power
\[ m^8 \]

4. 8 subtracted from a number
\[ n - 8 \]

5. Six times a number increased by 9
\[ 6 \cdot q + 9 \]

6. The difference of 7 and 3 times a number
\[ 7 - 3 \cdot m \]

7. 17 less than a number cubed
\[ a^3 - 17 \]

8. A number cut in half decreased by 5
\[ \frac{c}{2} - 5 \]

9. The product of 4 and a number squared
\[ 4 \cdot h^2 \]

10. \( \frac{x^3}{2} - 8 \)

11. 5 times the quantity \( x + y \)
\[ 5(x + y) \]
Translate the following algebraic expressions. **Show work** for full credit.

1. The product of 7 and a number  
   \[ 7 \cdot n \]

2. The difference of \( x \) and 5  
   \[ x - 5 \]

3. 14 increased by a number cubed  
   \[ 14 + c^3 \]

4. A number split up into 5 groups  
   \[ a \div 5 \]

5. 6 less than the product of 10 and a number  
   \[ 10 \cdot a - 6 \]

6. Seven times the quantity of \( a \) plus \( b \)  
   \[ 7 \cdot (a + b) \]

7. A number squared divided by 15  
   \[ \frac{x^2}{15} \]

8. Fifteen subtracted from triple a number  
   \[ 3x - 15 \]

9. The quotient of 4 and a number to the fifth power  
   \[ 4 \div n^5 \]

10. \( 2(a - b) \)

   Two times the quantity of \( a \) minus \( b \)

11. \( 7 - 3x^2 \)

   Seven minus the product of Three and a number squared.
Fill in the table and write an expression to represent each situation.

12. Darren saved twice the amount of money Jesse did. If Jesse saved \( m \) dollars, how much did Darren save?

<table>
<thead>
<tr>
<th>Jesse</th>
<th>Darren</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5</td>
<td>$10</td>
</tr>
<tr>
<td>$12</td>
<td>$24</td>
</tr>
<tr>
<td>( m )</td>
<td>( m \cdot 2 )</td>
</tr>
</tbody>
</table>

Expression: \( m \cdot 2 \)

13. George read a third of the pages Bill read for his summer reading project. If Bill read \( p \) pages, how many pages did George read?

<table>
<thead>
<tr>
<th>Bill</th>
<th>George</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>( p )</td>
<td>( p \div 3 )</td>
</tr>
</tbody>
</table>

Expression: \( p \div 3 \)

14. The width of a box is 4 inches less than the length. If the width of the box is \( x \) inches, how long is the length?

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>( x + 4 )</td>
<td>( x )</td>
</tr>
<tr>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

Expression: \( x + 4 \)

15. Simon spent \$12 per ticket for him and his friends to go to a concert. If Simon bought \( t \) tickets, how much money would Simon spend?

<table>
<thead>
<tr>
<th>Tickets</th>
<th>Money Spent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( \times ) 12</td>
<td>$36</td>
</tr>
<tr>
<td>7 ( \times ) 12</td>
<td>$84</td>
</tr>
<tr>
<td>( t ) ( \times ) 12</td>
<td>( t \times 12 )</td>
</tr>
<tr>
<td>2</td>
<td>$144</td>
</tr>
</tbody>
</table>

Expression: \( t \times 12 \)
Simplifying Algebraic Expressions

1) \(-3 + \frac{10}{b} \quad \text{use } b = 5\)
   \[-1\]
   \[\text{use } d = 32\]
   \[-\frac{d}{8} + 5\]
   \[1\]

2) \(-5 + \frac{w}{5} \quad \text{use } w = 10\)
   \[-3\]
   \[\text{use } n = 4\]
   \[3n + n\]
   \[16\]

3) \(6h - 9h \quad \text{use } h = 5\)
   \[-15\]
   \[\text{use } z = 6\]
   \[-8(9z - 6)\]
   \[-384\]

4) \(h + 5h \quad \text{use } h = 9\)
   \[54\]
   \[\text{use } b = 5\]
   \[-3(7 + 6b)\]
   \[-111\]

5) \(4(7 - 9d) \quad \text{use } d = 4\)
   \[\text{use } x = 7\]
   \[5x - 4x + 6\]
   \[13\]
Simplifying Algebraic Expressions

1) \(-7(-9r + 5h)\) use \(r = 9\) and \(h = 8\)
   \[287\]

6) \(9(-6c - 4b)\) use \(c = 6\) and \(b = 3\)
   \[-432\]

2) \(8w - 4 + 6x\) use \(w = 9\) and \(x = 4\)
   \[92\]

7) \(\frac{-16}{s} - 5 + 7c\) use \(s = 4\) and \(c = 7\)
   \[40\]

3) \(2 - \frac{18}{z} + 4n\) use \(z = 6\) and \(n = 4\)
   \[15\]

8) \(-8 - \frac{s}{9} + 6c\) use \(s = 27\) and \(c = 4\)
   \[13\]

4) \(4(-7c + 2d)\) use \(d = 4\) and \(c = 3\)
   \[-52\]

9) \(7 - 3s + 9k\) use \(s = 8\) and \(k = 7\)
   \[46\]

5) \(2c + 8x + 4\) use \(c = 8\) and \(x = 6\)
   \[68\]

10) \(4d - 6c - 7\) use \(d = 9\) and \(c = 5\)
    \[-1\]

Math-Aids.Com
Ordered Pairs

Tell what point is located at each ordered pair.
1. \((3, -2)\)   B
2. \((2, 3)\)   D
3. \((-5, 5)\)   O
4. \((-7, -8)\)   H
5. \((-4, 4)\)   C
6. \((-5, 0)\)   F

Write the ordered pair for each given point.
7. \(E (-3, -2)\)
8. \(M (1, -6)\)
9. \(P (8, 0)\)
10. \(G (7, 8)\)
11. \(Q (-8, 0)\)
12. \(N (5, 5)\)

Plot the following points on the coordinate grid.
13. \(S (-6, -3)\)
14. \(T (2, -4)\)
15. \(U (5, 0)\)
Plot the Points in the Grid

Plot the following points in the given grid:

A (1, 2)  B (-1, 5)  C (2, -3)  D (-4, -6)
E (0, 4)  F (6, 0)  G (0, 0)  H (0, -7)
Identify the Quadrants

Plot the following points and identify the quadrant in which they lie:

A (2, -6) \text{ Quadrant IV} \quad B (-7, -3) \text{ Quadrant III} \quad C (-5, 5) \text{ Quadrant II}

D (-4, -1) \text{ Quadrant III} \quad E (1, -1) \text{ Quadrant IV} \quad G (6, 4) \text{ Quadrant I}