Comparing Bits & Pieces UNIT
Main Focus: Ratios, Rational Numbers, and Equivalence

Factors and Multiples: Understand fractions and decimals as numbers that can be located on the number line, compared, counted, partitioned, and decomposed.

- Expand interpretations of a fraction to include expressing a fraction as a part–whole relationship, as a number, and as an indicated division
- Reason about the roles of the numerator and denominator in each of the interpretations of a fraction
- Use multiple interpretations of proper fractions, improper fractions, and mixed numbers
- Use decimals to represent fractional quantities with attention to place value
- Recognize that fractions are called rational numbers and that rational numbers are points on the number line
- Use the number line to reason about rational number relationships
- Use benchmarks to estimate the values of fractions and decimals and to compare and order fractions and decimals
- Recognize that fractions can represent both locations and distances on the number line
- Recognize that a number and its opposite are at equal distances from zero on the number line; the opposite of $a$ is $-a$ and the opposite of $-a$ is $a$
- Recognize that the absolute value of a number is its distance from 0 on the number line and use that value to describe real-world quantities
- Introduce percent as a part–whole relationship in which the whole is not necessarily out of 100, but is scaled or partitioned to be “out of 100” or “per 100”
- Apply a variety of partitioning strategies to solve problems

Ratios as Comparisons: Understand ratios as comparisons of two numbers.

- Use ratios and associated rates to compare quantities
- Distinguish between a difference, which is an additive comparison, and a ratio, which is a multiplicative comparison
- Distinguish between fractions as numbers and ratios as comparisons
- Apply a variety of scaling strategies to solve problems involving ratios and unit rates
- Recognize that a unit rate is a ratio in which one of the quantities being compared has a value of 1; use rate language in the context of a ratio relationship
- Scale percents to predict new outcomes

Equivalence: Understand equivalence of fractions and ratios, and use equivalence to solve problems.

- Recognize that equivalent fractions represent the same amount, distance, or location; develop strategies for finding and using equivalent fractions
- Recognize that comparing situations with different-sized wholes is difficult without some common basis of comparison
- Use partitioning and scaling strategies to generate equivalent fractions and ratios and to solve problems
- Develop meaningful strategies for representing fraction amounts greater than 1 or less than $-1$ as both mixed numbers and improper fractions
- Recognize that equivalent ratios represent the same relationship between two quantities; develop strategies for finding and using equivalent ratios
- Build and use rate tables of equivalent ratios to solve problems
Find the Ratios

= 42 : 6 = 7 : 1

= 56 : 64 = 7 : 8

= 54 : 36 = 3 : 2

= 30 : 45 = 2 : 3
Solve each problem.

Ex) At the burger shop the ratio of regular sodas sold to diet sodas sold was 3:6. For every _____ diet sodas sold there are _____ regular sodas sold.

1) For every 2 males birds in a bird cage there are 5 females. What is the ratio of males to females?

2) At the store the ratio of books sold to movies sold was 8:2. For every _____ books sold there were _____ movies sold.

3) At the pet store the ratio of dogs to cats was 4:7. For every _____ dogs there are _____ cats.

4) During the class election the ratio of votes for Tiffany to votes for Jerry was 4:3. For every _____ votes Jerry got Tiffany got _____.

5) In a bag of candy for every 9 chocolate pieces there are 6 sugar pieces. What is the ratio of chocolate pieces to sugar pieces?

6) For every 6 green apples in an orchard there were 9 red apples. What is the ratio of green apples to red apples?

7) At the movie theater the ratio of small popcorns sold to large popcorns sold was 5:9. For every _____ large popcorns sold there are _____ small popcorns sold.

8) The ratio of pickles to onions on a burger was 2:4. For every _____ pickles there are _____ onions.

9) For every 5 cars in a parking lot there are 6 trucks. What is the ratio of cars to trucks in the parking lot?

10) At an ice cream shop the ratio of chocolate cones sold to vanilla cones cones sold was 4:3. For every _____ vanilla cones sold there were _____ chocolate cones sold.

11) For every 4 hamburgers sold at the malt shop there are 2 hotdogs sold. What is the ratio of hotdogs sold to hamburgers sold?

12) For every 8 Wii games Janet owned she had 7 PS3 games. What is her ratio of Wii games to PS3 games?
Equivalent Ratios

Write two equivalent ratios.

1) \[
\begin{array}{ccc}
7 & 14 & 21 \\
9 & 18 & 27 \\
\end{array}
\]

2) \[
\begin{array}{ccc}
5 & 10 & 15 \\
8 & 16 & 24 \\
\end{array}
\]

3) \[
\begin{array}{ccc}
5 & 10 & 15 \\
7 & 14 & 21 \\
\end{array}
\]

4) \[
\begin{array}{ccc}
2 & 4 & 6 \\
11 & 22 & 33 \\
\end{array}
\]

5) \[
\begin{array}{ccc}
9 & 18 & 27 \\
4 & 8 & 12 \\
\end{array}
\]

6) \[
\begin{array}{ccc}
6 & 12 & 18 \\
7 & 14 & 21 \\
\end{array}
\]

Determine whether the ratios are equivalent.

7) \( \frac{4}{3} \) and \( \frac{20}{15} \) Yes

8) \( \frac{7}{6} \) and \( \frac{4}{7} \) No

9) \( \frac{8}{3} \) and \( \frac{4}{5} \) No

10) \( \frac{7}{8} \) and \( \frac{21}{24} \) Yes

11) \( \frac{11}{7} \) and \( \frac{5}{4} \) No

12) \( \frac{11}{4} \) and \( \frac{33}{12} \) Yes

Use equivalent ratios to find the unknown value.

13) \( \frac{b}{66} = \frac{5}{11} \) \( b = 30 \)

14) \( \frac{b}{54} = \frac{2}{9} \) \( b = 12 \)

15) \( \frac{12}{11} = \frac{v}{33} \) \( v = 36 \)

16) \( \frac{8}{9} = \frac{48}{s} \) \( s = 54 \)

17) \( \frac{22}{n} = \frac{11}{2} \) \( n = 4 \)

18) \( \frac{7}{4} = \frac{v}{28} \) \( v = 49 \)
Equivalent Ratios

1) Which two recipes have equivalent ratios of cups of flour needed to the number of cookies? **Oatmeal Raisin and Macadamia Nut**

<table>
<thead>
<tr>
<th>Recipes</th>
<th>Cups of Flour Needed</th>
<th>Number of Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sugar</td>
<td>3</td>
<td>46</td>
</tr>
<tr>
<td>Oatmeal Raisin</td>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>Macadamia Nut</td>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>Chocolate Chip</td>
<td>4</td>
<td>48</td>
</tr>
</tbody>
</table>

2) Which two recycling plants have equivalent ratios of green bottles to the total number of bottles recycled in one day? **Plant A and Plant D**

<table>
<thead>
<tr>
<th>Recycling Plants</th>
<th>Green Bottles Recycled</th>
<th>Total Bottles Recycled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant A</td>
<td>32</td>
<td>256</td>
</tr>
<tr>
<td>Plant B</td>
<td>18</td>
<td>140</td>
</tr>
<tr>
<td>Plant C</td>
<td>24</td>
<td>196</td>
</tr>
<tr>
<td>Plant D</td>
<td>13</td>
<td>104</td>
</tr>
</tbody>
</table>

3) Which two types of cars have equivalent ratios of miles traveled to hours of time during the trip? **Dodge and Chevrolet**

<table>
<thead>
<tr>
<th>Cars</th>
<th>Miles Traveled</th>
<th>Hours of Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford</td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>Dodge</td>
<td>64</td>
<td>4</td>
</tr>
<tr>
<td>Chevrolet</td>
<td>48</td>
<td>3</td>
</tr>
<tr>
<td>Toyota</td>
<td>25</td>
<td>2</td>
</tr>
</tbody>
</table>
"I Understand How Percents and Ratios are Related and Can Use Percents to Solve Real-World Problems."

Finding the Whole or Part in a Percent Proportion

A percent can always be used to compare a part to a whole. In a percent, the whole is always 100.

\[
\frac{\%}{100} = \frac{\text{part}}{\text{whole}}
\]

\[
\frac{\%}{100} = \frac{\text{is}}{\text{of}}
\]

What's the Difference?

25% of 40 is what number?

\[
\frac{25}{100} = \frac{x}{40} \Rightarrow 100x = 1000 \Rightarrow x = 25
\]

9 is 36% of what number?

\[
\frac{36}{100} = \frac{9}{x} \Rightarrow 36x = 900 \Rightarrow x = 25
\]

15 is what percent of 25?

\[
\frac{15}{25} = \frac{x}{100} \Rightarrow 1500 = 25x \Rightarrow x = 60\%
\]

1. 7 is 35% of what number?

\[
\frac{35}{100} = \frac{7}{x} \Rightarrow 7x = 3500 \Rightarrow x = 500
\]

3. 225% of 300 is what number?

\[
\frac{225}{100} = \frac{x}{300} \Rightarrow 225x = 67500 \Rightarrow x = 675
\]

2. 17 is what percent of 20?

\[
\frac{17}{20} = \frac{x}{100} \Rightarrow 1700 = 20x \Rightarrow x = 85
\]

4. 75 is 15% of what number?

\[
\frac{15}{100} = \frac{75}{x} \Rightarrow 15x = 7500 \Rightarrow x = 500
\]
Converting Improper Fractions to Mixed Numbers

1) \( \frac{72}{10} = 7 \frac{1}{5} \)  2) \( \frac{19}{8} = 2 \frac{3}{8} \)  3) \( \frac{37}{6} = 6 \frac{1}{6} \)

4) \( \frac{10}{3} = 3 \frac{1}{3} \)  5) \( \frac{36}{5} = 7 \frac{1}{5} \)  6) \( \frac{39}{5} = 7 \frac{4}{5} \)

7) \( \frac{13}{4} = 3 \frac{1}{4} \)  8) \( \frac{25}{4} = 6 \frac{1}{4} \)  9) \( \frac{25}{7} = 3 \frac{4}{7} \)

10) \( \frac{15}{6} = 2 \frac{1}{2} \)  11) \( \frac{22}{8} = 2 \frac{3}{4} \)  12) \( \frac{59}{8} = 7 \frac{3}{8} \)

13) \( \frac{77}{10} = 7 \frac{7}{10} \)  14) \( \frac{18}{8} = 2 \frac{1}{4} \)  15) \( \frac{50}{7} = 7 \frac{1}{7} \)

Converting Mixed Numbers to Improper Fractions

1) \( 9 \frac{1}{2} = \frac{19}{2} \)  2) \( 7 \frac{1}{4} = \frac{29}{4} \)  3) \( 6 \frac{1}{2} = \frac{13}{2} \)

4) \( 7 \frac{1}{7} = \frac{50}{7} \)  5) \( 3 \frac{2}{5} = \frac{17}{5} \)  6) \( 6 \frac{1}{6} = \frac{37}{6} \)

7) \( 6 \frac{1}{9} = \frac{55}{9} \)  8) \( 5 \frac{1}{4} = \frac{21}{4} \)  9) \( 8 \frac{9}{10} = \frac{89}{10} \)

10) \( 4 \frac{1}{3} = \frac{13}{3} \)  11) \( 6 \frac{1}{2} = \frac{13}{2} \)  12) \( 4 \frac{2}{5} = \frac{22}{5} \)

13) \( 9 \frac{1}{9} = \frac{82}{9} \)  14) \( 4 \frac{2}{9} = \frac{38}{9} \)  15) \( 3 \frac{2}{3} = \frac{11}{3} \)
1) $\frac{23}{100}$  
   a) 2300% b) 230% c) 0.23 d) 0.023  

2) $\frac{79}{100}$  
   a) 7.9 b) 79% c) 0.079 d) 790%  

3) $\frac{9}{10}$  
   a) 90% b) 90.0 c) 9% d) 0.09%  

4) $\frac{5}{100}$  
   a) 0.5 b) 50% c) 0.05 d) 500%  

5) $\frac{11}{100}$  
   a) 0.01 b) 11% c) 1.1 d) 110%  

6) $\frac{3}{10}$  
   a) 0.3 b) 0.03 c) 3% d) 300%  

7) $\frac{50}{100}$  
   a) 5% b) 0.05 c) 50% d) 0.005  

8) $\frac{7}{10}$  
   a) 7% b) 0.07 c) 7% d) 0.007  

9) $\frac{22}{10}$  
   a) 0.22 b) 2.2 c) 22% d) 2%  

10) $\frac{30}{100}$  
    a) 30% b) 0.03 c) 3.0 d) 3%  

11) $\frac{15}{100}$  
    a) 0.15 b) 1.5 c) 150% d) 1500%  

12) $\frac{33}{10}$  
    a) 33% b) 0.33 c) 0.03 d) 330%  

13) $\frac{20}{100}$  
    a) 200% b) 0.02 c) 20% d) 0.002  

14) $\frac{47}{100}$  
    a) 0.47 b) 0.47 c) 470% d) 4700%  

Printable Math Worksheets @ www.mathworksheets4kids.com
1. Use the number line to compare 7 and -2.

Which inequality compares 7 and -2 correctly?
- A. 7 > -2
- B. 7 < -2

7 is greater than -2

2. Which inequality compares -3 and -1 correctly?
- A. -3 < -1
- B. -3 > -1

-3 is less than -1

3. One day last winter, the temperature was -7°C at 8 A.M. The temperature was -10°C at 5 P.M. Which inequality compares -7 and -10 correctly?
- A. -7 > -10
- B. -7 < -10

-7 is greater than -10

4. Last night, some friends played a board game. They played two rounds. In the first round, one player lost 16 points. In the second round, the same player gained 13 points. Which inequality compares the player's points in the two rounds?
- A. 16 > -13
- B. -16 < -13
- C. -16 > 13
- D. 16 < -13

\[ \text{lost } 16 \rightarrow -16 \]
\[ \text{gained } 13 \rightarrow 13 \]

-16 is less than 13

5. The table shows the number of yards gained or lost by a football team on its first play with the ball in each of five games. Positive numbers represent yards gained. Negative numbers represent yards lost.

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>-12 yards</td>
<td>12 yards</td>
<td>-8 yards</td>
<td>9 yards</td>
<td>4 yards</td>
</tr>
</tbody>
</table>

a) Which list shows the numbers ordered from greatest to least?
- A. -8, 4, 12, -12, 9
- B. 12, 4, -12, -8, 9
- C. -12, -8, 4, 9, 12
- D. 12, 9, 4, -8, -12

b) In which game did the team do best on its first play?

Game 2 (gained 12 yards)
6. Five friends played a board game. The table shows their scores at the end of the game.

<table>
<thead>
<tr>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person</td>
</tr>
<tr>
<td>Score</td>
</tr>
</tbody>
</table>

a) Which list shows the numbers ordered from least to greatest?
- A. 9, 8, 7, -2, -3
- B. -3, -2, 7, 9, 8
- C. -2, 7, 9, -3, 8
- D. 9, 7, -3, -2, 8

b) If the person with the highest score won, who finished in second place?

Daniel

7. Multiple Representations Use the number line to compare 13 and -6.

Which inequalities compare 13 and -6 correctly? Check all that apply.
- A. 13 < -6
- B. 13 > -6
- C. -6 > 13
- D. 13 = -6
- E. -6 < 13

13 is greater than -6

8. Error Analysis Lucinda and Marco compare 4 and -2. Lucinda claims that 4 is to the right of -2 on a number line, and so 4 > -2. Marco claims that 4 is to the left of -2 on a number line, and so 4 < -2. Who is incorrect and what is the error?
- A. Lucinda is incorrect. Because 4 is to the right of -2 on a number line, the inequality should be 4 < -2.
- B. Marco is incorrect. Because 4 is to the left of -2 on a number line, the inequality should be 4 > -2.
- C. Marco is incorrect. On a number line, 4 is not to the left of -2.
- D. Lucinda is incorrect. On a number line, 4 is not to the right of -2.

9. Frosting Points A scientist has two different liquids in glass jars. Liquid A freezes at a temperature of -14°C. Liquid B freezes at a temperature of -7°C.

a) Write an inequality to compare these two temperatures.

-14°C < -7°C

b) Which liquid freezes at a colder temperature? Liquid A

10. Reasoning Sunday at noon, the temperature was 5 degrees above zero.

Wednesday at noon, the temperature was 7 degrees above zero.

a) Which inequality compares the two integers for this situation correctly?
- A. 5 < 7
- B. -5 < 7
- C. 5 > -7
- D. 5 < -7
- E. 5 > 7
- F. -5 > -7
- G. -5 > 7
- H. -5 < -7
Absolute Value

The **absolute value** of a number is its distance from 0 on the number line. Since absolute value is a distance, an absolute value is never negative. The symbol for the absolute value of a number \( n \) is \( |n| \).

**Example**

Find each absolute value.

a. \( |9| \) 9  
   b. \( |-3| \) 3  
   c. \( |0| \) 0  
   d. \( \left| -\frac{2}{7} \right| \) \( \frac{2}{7} \)

e. \( |-27| \) 27  
   f. \( |-27| \) 27  
   g. \( |-27| \) 27

**Example**

Order the values from greatest to least.

\[ \begin{array}{cccccc}
-7 & -3 & 3 & -1 & 0 & 7 \\
\end{array} \]

| Greatest | -7 | -3 | 3 | -1 | 0 | Least |

**Got it?**

Order the values from least to greatest:

\( |-5|, -5, |2|, -2, -|3|, -|3| \)

5, -5, 2, -2, -3, 3

\(-5, -|3|, -2, 12, |-3|, 1-5\)
Example

Compare each pair.

a. $|1.25| < 1\frac{1}{2}$

b. $-\frac{1}{3} < |0.67|

c. $1\frac{1}{2} = -1\frac{1}{2}$

d. $-1\frac{1}{2} < |1.8|

e. $\frac{1}{2} > -\frac{1}{3}$

f. $|1.8| > 10$

Plot the following rational numbers on the number line.

$-1.4, \frac{5}{2}, -3\frac{1}{2}, 4.065, \frac{9}{10}, 1\frac{2}{3}, -|0.25|, -\frac{20}{4}$

$\uparrow \uparrow$

$2\frac{1}{2} 3\frac{1}{2}$

$-0.25 -5$